Applying three VaR approaches in measuring market risk of stock portfolio: The case study of VN-30 stock basket in HOSE

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ABSTRACT

This study examines and applies the three statistical value at risk models including variance-covariance, historical simulation, and Monte Carlo simulation in measuring market risk of VN-30 portfolio of Ho Chi Minh stock exchange (HOSE) in Vietnam stock market and some back-testing techniques in assessing the validity of the VaR performance in the timeframe of January 30, 2012–February 26, 2016. The models are constructed from two volatility methods of stock price: SMA and EWMA throughout the five chosen confidence level: 90%, 93%, 95%, 97.5%, and 99%. The findings of the study show that the differences among the results of three models are not significant. Additionally, three VaR (Value at Risk) models have generally the similar accepted range assessed in both types of back-tests at all confidence levels considered and at the 97.5% confidence level. They can work best to achieve the highest validity level of results in satisfying both conditional and unconditional back-tests. The Monte Carlo Simulation (MCS) has been considered the most appropriate method to apply in the context of VN-30 portfolio due to its flexibility in distribution simulation. Recommendations for further research and investigations are provided accordingly.

1. Introduction

Risk management is a crucial concern in many institutions and countries around the world. The financial crises have exposed the uncertainty to investors' portfolios. The movements of stock price, exchange rate, interest rate, and commodity price are the sources of market risk that may cause potential losses to portfolio's value (Jorion, 2001). According to Duda and Schmidt (2009), many banks and institutions have been taking significant impacts in measuring market risk to set up an adequate capital base for their activities. Frain and Meegan (1996) had the same point of view as laying out several losses in banks and corporations in the US. Hence, the need for a suitable market risk measurement tool that can measure and set up an adequate capital base reserve as a cushion against potential losses is important. Cassidy and Gizycki (1997) assumed that value at risk (VaR), nowadays, is a widely used technique in measuring market risk. VaR measures the potential loss that is likely to occur if adverse movements in the market happen. VaR has become a standard measure for financial analysts to quantify market risk and accurately measure the high changes in prices due to three key characteristics: a specified level of loss, a fixed period of time, and a confidence level (Angelovska, 2013).

Vietnam stock market has been developing and popular with insider trading, herding behavior, and many inexperienced individual investors that would create more market risk to the players. In such context, most investors have been suffering huge losses due to their ignorance of potential losses. Finding a suitable risk measurement model is crucial for supporting the market risk management of investors, especially for organizational investors in Vietnam stock market. Therefore, this study attempts to test the appropriateness of the three basic VaR models, including variance-covariance method, historical simulation method, and Monte Carlo simulation method in the context of Vietnam stock market. In this study we used the VN-30 stock portfolio to find the portfolio's market risk and examine the differences among the three VaR models. Moreover, we also adopt some basic back-testing methods to test the accuracy and validity of the three models. The VN-30 stocks basket of HOSE is chosen because it contains top 30 highest capitalization stocks (around 80% of HOSE) with the trading volume of around 60% of HOSE, and attracts attention of both local and foreign investors. The research findings provide empirical evidence of the applicability of the three VaR models to measure market risk in a weak form capital market as Vietnam stock market, and reaffirm the accuracy and validity of the three models.

2. Literature review

2.1. Definition

Value at risk (VaR) is a method of measuring the maximum potential loss of the portfolio in a specific period of time in relative with a confidence level, or it can be described as the minimum potential loss that the portfolio will be exposed to in a given level of significance (Jorion, 2001). For instance, assuming that the initial portfolio value is *V(0)*, the current value of the portfolio is *V*, and the chosen confidence level is 95%, *VaR(95%)* is the amount of loss in which *P[V-V(0) <- VaR(95%)]* = 5% (Dowd, 2002).

VaR approaches are adopted based on the assumption of normality, with the general formula of VaR as follows:

$$VaR(1-\alpha) = MV*Z(1-\alpha)*\sigma$$

where $1-\alpha$ is the level of confidence and Z is the standard normal statistical value relative to $1-\alpha$.

Thus, theoretically, VaR has three parameters: (i) a specified level of loss: the risk exposure amount of current portfolio; (ii) a fixed period of time: a time frame considered to estimate the loss over; and (iii) a confidence level: the proportion of days covered by VaR amount (Angelovska, 2013).

In this study we choose the one day holding period to measure VaR with the confidence levels of 90%, 93%, 95%, 97.5%, and 99% to lay out the acceptable and valid range from back-testing for VaR models.

2.2. Value-at-risk approaches

According to Saita (2007), there are three main alternatives of VaR approaches that are mostly used for measuring market risk: (i) variance-covariance approach; (ii) historical simulation; and (iii) Monte Carlo simulation.

2.2.1. Variance-covariance (VC) method

It is the simplest method in VaR calculations, employed on the assumption of market normality (Wiener, 1999). The process of this method includes a mapped portfolio converted from original assets portfolio to contain only asset risk factors (as with stock portfolio, stock returns will be the risk factor), and a variance-covariance matrix or correlation matrix in presenting the relationship of risk factors. The general figure of the conversion between original stock portfolio and mapped portfolio is reported in Table 1.

Table 1

The mapping process of a portfolio

Original Portfolio (1)	Mapped Portfolio (2)
Stock 1	R1
Stock 2	R2
…..	…..
Stock n	Rn

The general formula of variance-covariance method in determining value at risk of the portfolio could be expressed as:

$$VARp = \sqrt{\overline{V} * C * \overline{V}^T}$$

where $\bar{V}$ (VaR vector) and $\overline{V}^T$ (the transposed vector of VaR vector) of the model would be as follows:

$$\bar{V}=\begin{bmatrix} Var\ 1 \\ Var\ 2 \\ \dots. \\ Var\ n-1 \\ Var\ n \end{bmatrix} \quad \text{and} \quad \overline{V}^{T} = [Var1 \quad Var2 \quad \dots.\ Var\ n-1 \quad Var\ n]$$

The correlation matrix would be denoted as symbol C, in which

$$C = \begin{bmatrix} 1 & \rho 1{,}2 \cdots & \rho 1, n \\ \vdots & \ddots & \vdots \\ \rho 1, n & \cdots & 1 \end{bmatrix}$$

It can be transformed into a more specific formula as:

$$VARp = \sqrt{\sum_{i=1}^{n}(VARi^2) + \sum_{i=1}^{n}\sum_{j\neq i}^{n}(VARi * VARj * \rho i, j)}$$

with $i, j = 1, 2, 3 \dots n\ (i \neq j)$

However, besides the simple implementation of use, the variance-covariance method can be inappropriate for the empirical distribution. As usually indicated by other researchers, the empirical distributions typically have fat tail relative to normal distribution, and hence the actual loss results usually have greater values as the normal estimation.

2.2.2. Historical simulation (HS) method

The historical simulation is a non-parametric method with no data distribution assumption needed (Hendricks, 1996). For a stock portfolio, this method simply creates a hypothetical time series of returns of the portfolio with the current weights of compositions invested in the portfolio. The maximum potential loss VaR can be directly found at the desired confidence level by using historical changes. However, these portfolio changes are not real historical returns of the portfolio, but the returns that the portfolio would have been experienced if the assets weights remain constant over time.

This method is easy to implement, assuming that historical data would be a good proxy for future measurement. Hence, it would capture all the empirical events, and the risk of the portfolio would likely be in the past (Rob van den Goorbergh & Vlaar, 1999). Still, the method has many limitations if used like the availability of data sources or the time frame to measure. The historical data could become a wrong indicator because the changing volatility and correlation through time could cause one to ignore the potential risk of extreme market movements (Allen et al., 2004).

2.2.3. Monte Carlo Simulation (MCS) method

The Monte Carlo Simulation was developed to overcome the limitations of historical simulation by having the ability to generate additional observations that are consistent with the recent historic events to bring the distribution of data into a normal distribution and finding VaR with the relative desired percentile as the historical simulation (Sanders & Cornett, 2008). Thus, the idea of this method is the central limit theorem in which if we have sufficiently large observations of data, our distribution would be approximated to the normal distribution (Anderson et al., 2011). This method is often used for finding VaR of complex portfolios as multi-risk factors portfolios or non-linear correlated risk factors portfolio (like options).

The simple process of the MCS for one risk-factor portfolio assumed as a one-stock portfolio, which followed Jorion (2001) and Alexander (2005), is simply to simulate the value of that stock with the random standard normal variables $Z \sim N(0,1)$, which is derived from many draws of random numbers between 0 and 1. The simulation will be repeated as many times as possible (preferably 10000 times), with each simulation measured over a period time T, in which the time T is divided into N small incremental times ΔT and the value simulated of the $k\Delta T$ period is the compounding of the $(k-1)\Delta T$ simulated value (with $k=1 \dots N$ and starting the initial value $S(0)$ invested in that stock). The general formula of this process is presented as:

$$S(k\Delta T) = S(0) * \prod_{k=1}^{N} exp\,[\mu * \Delta T + Z(k\Delta T)\,\sigma * \sqrt{\Delta T}];$$

This is the Brownian motion process in which μ is the mean return of the stock and σ is the standard deviation of the stock.

Nevertheless, for the case of multiple stocks portfolio, the correlated factors of stocks components should be included to truly reflect the simulation. Hence, in accordance with Best (1998), Allen et al. (2004), Alexander (2008), and Dowd (2005), the value of each stock in the multiple-stocks portfolio can be simulated by the correlated random standard normal variable Φi, as follow:

$$Si(k\Delta T) = Si(0) * \prod_{k=1}^{N} exp\,[\mu * \Delta T + \Phi i(k\Delta T)\,\sigma * \sqrt{\Delta T}]$$

where $Si(0)$ is the initial weighted investment of the current portfolio in that stock, $Si(k\Delta T)$ is the simulated stock price at a specific $k(th)\Delta T$, and $\Phi i(k\Delta T) = A * \begin{bmatrix} Z1(k\Delta T) \\ Z2(k\Delta T) \\ \dots \\ \dots \\ Z30(k\Delta T) \end{bmatrix}$ with A being the Cholesky decomposition factor of correlation matrix C, in which $C = A * A^T$.

After we simulate each stock with those correlated random standard variables, we will sum all the simulated stocks' values to get the simulated portfolio's value.

Yet, besides the fact that the Monte Carlo simulation has many advantages over the historical simulation, this method reveals many limitations such as the error in the size of time discrete (ΔT)—the Brownian motion process is continuous; thus, the smaller the size of ΔT (or the larger the size of N), the smaller the error—and the error from the number of simulation trials because the standard error will decrease (or the accuracy will increase) with the square root of the number of simulations.

2.2.4. Discussion of three VaR models and previous findings

In some real market conditions, the variance-covariance and 10000 times Monte Carlo simulation have been suggested to be less efficient in estimating VaR because the actual data distributions mostly have fatter tails than the normal ones, and hence the actual losses would be most likely larger than are estimated. This is also the reason why banks and institutions have suffered great losses and gone bankrupt during the credit crunch. They, in fact, have underestimated the risk when looking at VaR based on normal market conditions. Thus, a VaR estimation based on the fat tail distribution was shown to have better forecast and measurement.

Many previous findings of Linsmeier and Pearson (1996), Alžbˇeta Holá (2012), Bohdalova (2007), Lupinski (2013), and Corkalo (2011) demonstrated the reliance of the VaR models in the market on the comparison between value at risk amount and the actual mark-to-market portfolio P/L with the two questions under consideration: (i) is the assumed distribution of the models consistent with the actual distribution of portfolio P/L? and (ii) does the number of actual losses exceed the VaR amount with expected frequency? For the first consideration, as indicated above, most authors generally maintain that their actual distributions have fatter tails than the normal ones, and hence the value of variance-covariance method as well as Monte Carlo simulation with a large number of times should be different from that of historical simulation. In addition, the second suggests that we conduct some back-tests to verify the models' accuracy and check the consistency of the frequency of losses exceeding VaR.

2.3. Back-testing methods

VaR models have many benefits in finding the market risk for our portfolio to set up a capital base. However, along with the benefits, there are many shortcomings of these models, hence raising concerns about the accuracy of the VaR estimated as well as the frequency of exceptions (Campbell, 2005). For this reason these risk models need to be regularly validated, and the back-testing methods are used to test the accuracy of these VaR models (Dowd, 2005). It should be essential to conduct as many tests as possible because the more tests there are to confirm that the model is being accepted, the more valid that model is. In theory, good VaR models are those that could capture the correct frequency of exceptions (or the failure rate) and could satisfy the independence of those exceptions (Finger, 2005) over the timeframe studied. The exceptions are those losses observed that have values greater than the VaR measured from the model. Hence, to follow up we conduct two main types of test: unconditional coverage and conditional coverage. The unconditional coverage test includes the Kupiec's proportion of failure test (POF test) and the time until first failure test (Tuff test) to check the consistency of actual exceptions frequency observed compared with the frequency suggested by the significance level. The conditional coverage test includes the independence test and joint test, which examine whether exceptions occurrences observed are independent from each other over time.

2.3.1. Unconditional coverage tests

2.3.1.1. Kupiec's proportion of failure test (POF test)

This test is conducted to examine whether the frequency of exceptions is in line with the model's significance level (Kupiec, 1995), which is α. We have the null hypothesis as:

H_0: $\alpha = x/T$

where x is the number of exceptions observed over the period of time T (x/T is the failure rate).

According to Jorion (2001), we will have our likelihood ratio calculation for this test as a statistical value as follows:

$$LR(pof)=-2*ln\left(\frac{(1-\alpha)^{T-x}*\alpha^{x}}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}*\left(\frac{x}{T}\right)^{x}}\right)$$

which is distributed with the chi square test (1 degree of freedom). We will accept the null hypothesis if the result of $LR(pof)$ < critical value of χ^2 distribution of a given confidence level of 1 degree of freedom.

2.3.1.2. Time until first failure test (Tuff test)

The idea of this test is to examine the failure rate defined by the time until first exceptions observed and whether it is in line with the suggested model's failure rate of the first exception (Kupiec, 1995). Let V be the time until the first exception. If our model suggests that α is the probability of having the exceptions in the time V, then we have our relative probability of the first exception suggested by the model as: $\alpha*(1-\alpha)^\wedge(V-1)$.

Regarding the test, we have the null hypothesis as follows:

H_0: $\alpha=1/V$

Moreover, with the likelihood ratio calculations as: $LR\ (tuff) = -2*ln\left(\frac{\alpha*(1-\alpha)^{v-1}}{\frac{1}{v}*(1-\frac{1}{v})^{v-1}}\right)$, distributed with chi square of 1 degree of freedom, we accept the model if the value of $LR(tuff)$ is smaller than the critical value of χ^2 distribution of a given confidence level with 1 degree of freedom.

2.3.2. Conditional coverage tests

2.3.2.1. Independence test

In this test our primary aim is to capture whether the occurrence of today's exception is dependent on the previous day's exception. This test is used to detect clustering problems in VaR measurements of the model. The clustering problems occur when the model could not adapt to the new situations of the market or the new volatilities and correlations.

We set up for this test a deviation indicator (It) and:

I(t)= 1 if VaR is exceeded; and

I(t) = 0 if VaR is not exceeded

Let *T(i,j)* be the number of days and assuming that condition *j* occurs today and that condition *i* occurs on the previous day. (*i* or *j* equals 0 or 1, depending on each case).

We construct a 2x2 contingency table of exception as follow:

Table 2

Contingency table of exceptions with conditional and unconditional occurrences

		Conditional		Unconditional
		Previous day		
j	*i*	*I(t-1)=0*	*I(t-1)=1*	SUM
Today	*I(t)=0*	*T(0,0)*	*T(1,0)*	*T(0,0)+T(1,0)*
	I(t)=1	*T(0,1)*	*T(1,1)*	*T(0,1)+T(1,1)*
		T(0,0)+T(0,1)	*T(1,0)+T(1,1)*	*N*

Let *pi,1* be the probability of having an exception today on the conditional state *i* occurring on the previous day:

$P(0,1)= \frac{T(0,1)}{T(0,0)+T(0,1)}$; $P(1,1)= \frac{T(1,1)}{T(1,0)+T(1,1)}$; $P= \frac{T(0,1)+T(1,1)}{N}$ with $N= T(0,0)+T(0,1)+T(1,0)+T(1,1)$

If the exception that occurs today is not dependent on the previous day occurrences:

$P(0,1)= P(1,1)= P$ (or the unconditional probability equals the conditional probability).

The relevant test statistics of independence is the likelihood ratio suggested by Christoffersen (1998) as:

$$LR(ind)= -2*ln\left(\frac{(1-p)^{T(0,0)+T(1,0)}*p^{T(0,1)+T(1,1)}}{[1-p(0,1)]^{T(0,0)}*p(0,1)^{T(0,1)}*[1-p(1,1)]^{T(1,0)}*p(1,1)^{T(1,1)}}\right)$$

distributed with chi square distribution of 1 degree of freedom.

This is the likelihood-ratio under the null hypothesis that the exceptions are independent across the days (Jorion, 2001). *LR(ind)* will be distributed with the chi square of 1 degree of freedom. Thus, we also conclude that the exceptions' occurrences are independent if the value of *LR(ind)* is smaller than the critical value of χ^2 distribution of a given confidence level with 1 degree of freedom.

2.3.2.2. Joint test

The joint test (Christoffersen, 1998) is the combination of POF tests [*LR(pof)*] and independence test [*LR(ind)*]. We have the conditional likelihood-ratio, *LR(cc)*, which captures both the frequency of VaR and independence of the exception as follows:

$LR\ (cc) = LR\ (pof) + LR(ind)$,

with chi square distribution with 2 degrees of freedom.

We accept the results if the value of *LR (cc)* is smaller than the critical value of χ^2 distribution of the given confidence level with 2 degrees of freedom.

3. Methodology

3.1. Data collection

The data of VN30 was collected for the whole 4 years from January 30, 2012 through February 26, 2016 (1016 days of timeframe). All the stock basket compositions of VN-30 during the timeframe were gathered. Sources of data and changing compositions of VN-30 are available on websites (hsx.vn, cafef.vn, vietstock.vn). We assumed that our portfolio investment would be VND100,000,000.

As a matter of fact, the VN-30 stock basket compositions change every six months due to the selection of new qualified stocks in the basket. However, a changing composition would make it hard to define the portfolio volatility throughout the whole period of time. Hence, we develop some specific assumptions of weights and stock components for our volatility and VaR calculations, and suggest denoting i to represent 30 positions in VN-30 and have stock 1,… , stock 30, accordingly. Each stock i has its own historical rate of returns in that position throughout the whole timeframe and the weight *i (Wi)* of that position. Each weight i is the average value of market capitalization proportions in that position throughout the timeframe. Thus, all the data of stocks feature changes in every six-month period at the position i in the timeframe, and the market capitalization value can be calculated in each period at that position with the changing stock price in that period. Then, we divide market capitalization at the position i for the sum of the whole basket's market capitalization to find the proportion of that stock in the basket at the position i across the time. Next, we take the average of those proportions' values through the whole timeframe at the position i to find the weight i (*Wi*) for investing in that relative position i of the portfolio.

Following the Resolution of HSX (2012) in choosing VN-30 stocks, we calculate the market capitalization of one stock as the product of its stock price, number of stock outstanding, the free float rate, and the limit percentage of market capitalization allowed of that stock in the basket. Hence, the general formula is:

*Market Cap. Of Stock = (price of stock) * (number of stock outstanding) * (free-float ratio) * (limit percentage of market capitalization allowed)*

By using this method we feasibly find the volatility of VN-30 portfolio during 2012–2016 through the correlation matrix of standard deviations between the stock i (i= 1,2…30). This method can work more accurately if the change in the market capitalization proportion at the specific position i throughout the time is not much.

3.2. Calculation process

The VaR measurement is performed using 5 confidence levels: 99%, 97.5%, 95%, 93%, and 90%. According to Nieppola (2009) and Dowd (2005), these confidence levels can enhance the power of the model in balancing type I and type II errors.

For the matter of interest, we employ two types of volatility in measuring VaR of VN-30: simple moving average (SMA) and exponential weighted moving average (EWMA).

First, SMA is based on the assumption of available observations with equal weights of volatility throughout the time: $\sigma = \sqrt{\frac{\sum_{x=1}^{n}(Rx-\bar{R})^2}{n-1}}$ for one day volatility, the SMA covariance between two assets is: $Cov(Ri,Rj)= \frac{\sum_{x=1}^{n}(Ri,x-\overline{Ri})*(Rj,x-\overline{Rj})}{n-1}$ (Saita, 2007).

Second, EWMA is adopted to assign more weights to the more recent volatility, which accurately reflects new changes' effects of market conditions:

$$\sigma(t,n) = \sqrt{\frac{\sum_{x=1}^{n} \lambda^{x-1} * R_{t-x}^2}{\sum_{x=1}^{n} \lambda^{x-1}}}$$

where $\sigma(t,n)$ is the volatility of the stock at time t with a sample of n returns and λ is the decay factor equaling 0.94 for one-day time horizon (RiskMetrics-Technical Document, 4e, 1996).

- Thus, the EWMA covariance between two assets is:

$$Cov(Ri, Rj)= \frac{(1-\lambda)}{(1-\lambda^n)} * \sum_{x=1}^{n} \lambda^{x-1} * R_{i,t-x} * R_{j,t-x}$$

Source: Saita (2007)

3.2.1. Variance-covariance method

We adopt this technique, taking the following steps:

- Transfer the original portfolio into a mapped portfolio which contains 30 stocks' returns as the risk factors.

- Find the standard deviation of each stock using SMA and EWMA approaches.

- Find covariance of stock returns from SMA volatility and EWMA volatility.

- Find standard deviation of the whole portfolio through covariance found from the two methods of volatility:

$$\sigma p = \sqrt{\sum_{i=1}^{30} \sum_{j=1}^{30} * wi * wj * Cov(Ri, Rj)};$$

i,j=(1,2,3…30).

- Find VaR of the portfolio: $VARp = MV*Z(1-\alpha)*\sigma p$, where *MV* is the current portfolio's value and $Z(1-\alpha)$ is the standard variable relative to confidence level $(1-\alpha)$.

3.2.2. Historical simulation

There are four steps to be followed:

- Collect the data and find the historical returns of stocks for the simulation for the period of 2012–2016:

$$Ri,t= Ln(Pi,t/Pi,(t-1))$$

where *Ri,t* is the return of the stock and *Pi,t* is the price of stock *i* at the end of day *t* (i= 1, 2, 3…30).

- Find the daily historical return of the whole portfolio:

$$Rp,t = \alpha p + \sum_{i=1}^{30} wi * Ri,t$$

where Rp,t is the return of the whole portfolio at the end of day t (i= 1, 2, 3…30).

- Run the simulation of historical daily changes of the portfolio by multiplying the total current value of the investment with each of historical portfolio's returns.

- Sort these historical value changes of the simulation in a descending order, create a distribution of value changes, and find the value at risk at the certain percentile desired.

3.2.3. Monte Carlo simulation

Given this technique, a few more steps are to be considered:

Find the initial weighted value investment for each stock:

$Si(0) = 100{,}000{,}000 * Wi;\ i=(1,2\ldots30)$

Find the mean returns of each stock in the timeframe chosen by day and its one-day standard deviation μ_i and σ_i.

As the larger the N, the better the model, we prefer to use N= 270 increments of 1 day, and divide the trading hours per day into minutes. The trading hours of stocks on HOSE are 9 a.m.–11:30a.m. and 13p.m.–15p.m. (i.e. 4.5 hours per day or 270 minutes per day). Hence, we have $\Delta T = 1/270$.

Find the mean returns and standard deviation of each stock for one unit of incremental time ΔT:

$\overline{\mathrm{Ri}}(\Delta T) = \mu_i/270$; i= (1,2…30)

$\sigma_i(\Delta T) = \sigma_i * (1/\sqrt{270})$;

Estimate the correlation matrix C of the portfolio to define the Cholesky decomposition factor with $C = A*A^T$:

$A1,1 = \sqrt{C1,1};$

$Ai,1 = \frac{Ci,1}{A1,1}$; for i=2, 3…30;

$Ai,i = \sqrt{Ci,i - \sum_{p=1}^{i-1} Ai,p^2}$; for i= 2, 3,… 30;

$Ai,j = \frac{1}{Aj,j} * \left(Ci,j - \sum_{p=1}^{j-1} Ai,p * Aj,p\right)$; for $i > j$ and $j \geq 2$

We create an appropriate (30x30) matrix A to find the correlated random standard normal variable Φi.

Find the correlated random standard normal variable Φi for the relative 30 stocks with the matrix A, which has just been identified and a vector of random standard normal variables $Zi(k\Delta T)$, which is inversely derived from random numbers between 0 and 1, as follow

$$\Phi i\ (k\Delta T) = A * \begin{bmatrix} Z1(k\Delta T) \\ Z2(k\Delta T) \\ \ldots \\ \ldots \\ Z30(k\Delta T) \end{bmatrix} = \sum_{j=1}^{i} Ai,j * Zj(k\Delta T),$$

with $k = 1,2\ldots270$ and $i, j = 1, 2, 3\ldots30$

Then, we have the relative $\Phi1(k\Delta T)$, $\Phi2(k\Delta T)$ …$\Phi30(k\Delta T)$ for stock 1, stock 2…stock 30, respectively.

Repeat Step 6 with 270 times of draws (k=1,2…270) from a normal distribution of $N(0,1)$ to find the vector of standard normal variables $Zi(k\Delta T)$ for each unit of incremental time ΔT and create $\Phi i(k\Delta T)$ for 270 incremental times ΔT of one day simulation.

To create simulated stock returns which are normally distributed with the mean of $\overline{Ri}(\Delta T)$ and standard deviation of $\sigma_i(\Delta T)$, simulate the stock's value with the relative correlated random standard normal variable $\Phi i(k\Delta T)$ periodically:

$Ri(k\Delta T) = \overline{Ri}(\Delta T) + \sigma_i(\Delta T) * \Phi i(k\Delta T)$, with k= (1,2,3….270) and *Ri(k)* is the stock return generated with *Z(kΔT)*.

$$Si(k\Delta T) = Si(0) * \prod_{k=1}^{N} exp\ [\overline{Ri}(\Delta T) + \Phi i(k\Delta T)\ \sigma_i(\Delta T)].$$

Calculate the simulated portfolio value at the end of the day as the sum of 30 stocks' value investment simulated at k=270:

$$Vp = S1(270\Delta T) + S2(270\Delta T) + \ldots + S30(270\Delta T)$$

where *Vp* is the simulated portfolio value.

Repeat the simulation of 30 stocks in Step 8 with their relative correlated random standard normal variable $\Phi i(k\Delta T)$ for 10000 times to create 10000 scenarios of tomorrow's potential 30 stocks' values and find the relative 10000 simulated portfolio values as in Step 9.

Find the changes of each simulated scenario by (Vp - 100,000,000VND), and thus we have 10000 scenarios of tomorrow's changes in portfolio value.

Arrange these changes in an order and find VaR with the desired level of confidence, similarly defined with the historical simulation approach.

3.3. The back-testing process

For the back-tests, we employ each method to back-test the results of VaR models in order to accept or reject the model based on the critical value of statistical test introduced above and throughout 5 chosen confidence levels. In general, the process can be generalized as follows:

Select a significance level in order to estimate the critical value related to the null hypothesis being true.

Calculate the likelihood ratios of each method (or statistical value) and compare them to the relative critical value with relative degrees of freedom for 5 confidence levels.

If the result of ratios (or calculated statistical value) is larger than the critical value of significant level with chi-square distribution with relative degree of freedoms, the VaR result is rejected, or it is accepted otherwise.

4. Results and discussion

4.1. VaR results

In general, changes in the proportion of market capitalization at a specific position *i* through time *t* are not much. The weight *i* for each position *i* in the portfolio is calculated, and the results are as follow:

Table 3

Average weight distribution of Stock 1–Stock 30 from the highest to smallest weight

Stock	AVG. Weight	AVG. Return
1	12.200%	-0.008%
2	10.670%	0.065%
3	9.576%	0.026%
4	7.748%	0.093%
5	6.445%	0.012%
6	5.729%	0.104%
7	5.456%	-0.016%
8	4.639%	-0.054%
9	4.334%	0.005%
10	3.849%	0.049%
11	3.431%	0.137%
12	2.821%	0.090%
13	2.639%	-0.002%
14	2.468%	0.010%
15	2.108%	0.006%
16	1.997%	0.043%
17	1.784%	-0.011%
18	1.681%	0.151%
19	1.474%	0.072%
20	1.260%	0.044%
21	1.154%	0.065%
22	1.028%	0.039%
23	0.972%	0.075%
24	0.895%	0.051%
25	0.785%	0.131%
26	0.722%	-0.064%
27	0.654%	-0.048%

Stock	AVG. Weight	AVG. Return
28	0.586%	0.073%
29	0.505%	-0.124%
30	0.391%	-0.006%

Volatilities of the portfolio calculated using SMA and EWMA methods are presented in Tables 4 and 5 as below:

Table 4

Portfolio's SMA-volatility

Portfolio Variance (SMA)	Portfolio STD. (SMA)
0.000126812	0.011261077

Table 5

Portfolio's EWMA-volatility

Portfolio Variance (EWMA)	Portfolio STD. (EWMA)
0.000117506	0.010840036

The difference between the results of two portfolio's volatilities exists. However, the difference is very small, or the two results are approximately equal. Hence, it is suggested that the VaR results measured in the same model for a given confidence level would be the same under any of these types of volatility used, and similar back-test results are also obtained. The VaR results of 3 models throughout 5 confidence levels with 2 types of volatility are shown in Table 6.

Table 6

VaR results

	SMA			EWMA		
	Historical Simulation	Variance-Covariance	Monte Carlo Simulation	Historical Simulation	Variance-Covariance	Monte Carlo Simulation
99.00%	(3,305,720.87)	(2,619,718.35)	(2,563,666.35)	(3,305,720.87)	(2,521,769.48)	(2,445,310.79)
Exceptions (x)	10.00	22.00	22.00	10.00	22.00	25.00
x/T	0.01	0.02	0.02	0.01	0.02	0.02
97.50%	(2,434,051.12)	(2,207,130.62)	(2,120,234.22)	(2,434,051.12)	(2,124,608.02)	(2,047,539.13)
Exceptions (x)	25.00	31.00	34.00	25.00	34.00	40.00

	SMA			EWMA		
	Historical Simulation	Variance-Covariance	Monte Carlo Simulation	Historical Simulation	Variance-Covariance	Monte Carlo Simulation
x/T	0.02	0.03	0.03	0.02	0.03	0.04
95%	(1,773,356.90)	(1,852,282.40)	(1,794,474.97)	(1,773,356.90)	(1,783,027.26)	(1,700,122.70)
Exceptions (x)	52.00	49.00	52.00	52.00	52.00	55.00
x/T	0.05	0.05	0.05	0.05	0.05	0.05
93%	(1,535,748.08)	(1,661,899.70)	(1,608,279.30)	(1,535,748.08)	(1,599,762.79)	(1,527,358.64)
Exceptions (x)	67.00	56.00	61.00	67.00	62.00	67.00
x/T	0.07	0.06	0.06	0.07	0.06	0.07
90%	(1,234,030.91)	(1,443,165.14)	(1,379,677.99)	(1,234,030.91)	(1,389,206.52)	(1,322,558.00)
Exceptions (x)	101.00	77.00	81.00	101.00	80.00	91.00
x/T	0.10	0.08	0.08	0.10	0.08	0.09

In Table 6, from 90% to 99% confidence levels, we find that our VaR results generated from the three models are not significantly different from each other for a given confidence level, especially for the variance-covariance and Monte Carlo simulation results. However, at the 95% confidence level, all the models' results are approximately similar, but the difference among them becomes larger as we move further away from the 95% point. As we move closely to 99%, the values of variance-covariance and Monte Carlo simulation are smaller than of the historical simulation. The opposite outcome occurs as we move closely to 90%. The reasons for this could be as follows:

The historical simulation is found from the real past movements of the portfolio, and hence the model's results obviously capture merely the same failure rate observed. Meanwhile, the variance-covariance and Monte Carlo simulation with 10000 simulations extract VaR from a normal condition distribution. For this reason the values of VC and MCS are the most approximate while the historical simulation has a little different record from the other two.

The actual distribution of the portfolio data has a positive kurtosis of (2.7323), having a fatter tail than the normal distribution. Thus, the 95% point seems to be intersection of the actual data distribution, and the normal data distribution is assumed because all the models' results are most approximate at this 95% confidence level. A fatter tail means greater actual losses at the left-end tail (95–99% and more) of the distribution comparing to the losses measured with normal market conditions.

4.2. Comparisons with previous findings

The similarity between our findings and those of previous studies is that the distribution of the empirical data has a positive kurtosis, i.e. a fatter tail than a true normal distribution. As a result, the VaR of historical simulation is higher than those estimated by both the variance-covariance and Monte Carlo simulation methods as the confidence level corresponds to the left-end tail (close to 99%) of the distribution.

However, the difference of our findings from those of earlier investigation is that it is more likely for the VaR of the Monte Carlo simulation to approximate that of the variance-covariance method than that of the historical simulation. Some researchers have used 100 times of simulation; the Monte Carlo simulation's results, thus, approximate those of the historical simulation.

4.3. Back-test results

The back-tests are done for 5 chosen confidence level, including 99%, 97.5%, 95%, 93%, and 90%, throughout 3 models under both types of volatilities used. In general, the back-tests' results of 3 models from 2 methods of volatilities are similar. Below are the results of each one.

4.3.1. Kupiec's proportion of failure (POF) test

The POF test shows that the most acceptable range of the test is around the 95% confidence level and the rejected results of the variance-covariance and Monte Carlo simulation method at the 99% and 90% confidence levels. The historical simulation results are all accepted and presented in Table 7.

Table 7

POF test's results

	POF Test (SMA)			POF Test (EWMA)		
	Historical Simulation	Variance-Covariance	Monte Carlo Simulation	Historical Simulation	Variance-Covariance	Monte Carlo Simulation
99%						
LR(POF)	0.00255847	10.45361826	10.45361826	0.002558467	10.45361826	15.56090087
Critical value	6.6348966	6.634896601	6.634896601	6.634896601	6.634896601	6.634896601
Conclusion	Accepted	Rejected	Rejected	Accepted	Rejected	Rejected
97.50%						
LR(POF)	0.00649404	1.184475294	2.704450666	0.006494039	2.704450666	7.346670145
Critical value	5.02388619	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Rejected

	POF Test (SMA)			POF Test (EWMA)		
	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation
95%						
LR(POF)	0.02961839	0.067901198	0.029618393	0.029618393	0.029618393	0.356353943
Critical value	3.84145882	3.841458821	3.841458821	3.841458821	3.841458821	3.841458821
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
93%						
LR(POF)	0.26135833	3.710777531	1.62164719	0.261358332	1.310703	0.261358332
Critical value	3.28302029	3.283020287	3.283020287	3.283020287	3.283020287	3.283020287
Conclusion	Accepted	Rejected	Accepted	Accepted	Accepted	Accepted
90%						
LR(POF)	0.00394392	7.161285268	4.952349795	0.003943917	5.463561327	1.268913099
Critical value	2.70554345	2.705543454	2.705543454	2.705543454	2.705543454	2.705543454
Conclusion	Accepted	Rejected	Rejected	Accepted	Rejected	Accepted

In Table 7 there are still some unexpected accepted results of EWMA-MCS at 90% and rejected results of SMA-VC and of EWMA-MCS at 93% and 97.5% respectively. In general, nevertheless, the most trustful range of validity lies between 93% and 97.5% because most of the results of 3 models from both volatility methods are accepted.

4.3.2. Time until first failure test (Tuff test)

Table 8

Tuff test's results

	Tuff Test			Tuff Test		
	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation
99%						
1/V	0.034482759	0.034482759	0.034482759	0.034482759	0.034482759	0.034482759
Tuff (LR)	1.07345361	1.07345361	1.07345361	1.07345361	1.07345361	1.07345361
Critical value	6.634896601	6.634896601	6.634896601	6.634896601	6.634896601	6.634896601

	Tuff Test			Tuff Test		
	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
97.50%						
1/V	0.034482759	0.034482759	0.037037037	0.034482759	0.037037037	0.090909091
Tuff (LR)	0.095850586	0.134944633	0.140114136	0.095850586	0.140114136	1.182120926
Critical value	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
95%						
1/V	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091
Tuff (LR)	0.315336293	0.315336293	0.315336293	0.315336293	0.315336293	0.315336293
Critical value	3.841458821	3.841458821	3.841458821	3.841458821	3.841458821	3.841458821
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
93%						
1/V	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091
Tuff (LR)	0.067939789	0.067939789	0.067939789	0.067939789	0.067939789	0.067939789
Critical value	3.283020287	3.283020287	3.283020287	3.283020287	3.283020287	3.283020287
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
90%						
1/V	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091	0.090909091
Tuff (LR)	0.010386357	0.010386357	0.010386357	0.010386357	0.010386357	0.010386357
Critical value	2.705543454	2.705543454	2.705543454	2.705543454	2.705543454	2.705543454
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted

The results of the Tuff test are incredibly different from those of the Kupiec's POF Test. All the models at all levels of confidence are accepted. Furthermore, many conclusions (labeled as accepted) do not support effectively the observed (*1/V*): Even though the observed probability of the exceptions occurrence in the time *V* is different significantly from the suggested model's failure rate (α), the test still accepts it. According to Dowd (2005), this Tuff test may not work effectively for a long timeframe because it only uses the time until first exception as an input factor. Hence, the longer the timeframe, the more exceptions occurrences there are, and after that, we ignore the less suitability of the first exceptions probability with the failure rate. Given our case, it would not be suitable for VN-30 portfolio.

4.3.3. Independence test

Table 9

Independence test's results

	SMA- Independence Test			EWMA- Independence Test		
	Historical Simulation	Variance-Covariance	Monte Carlo Simulation	Historical Simulation	Variance-Covariance	Monte Carlo Simulation
99%						
LR(ind)	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	0.153882338
Critical value	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187
Conclusion						Accepted
97.5%						
LR(ind)	0.153882338	2.822118853	4.135493351	0.153882338	4.135493351	2.920802126
Critical value	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187	5.023886187
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
95%						
LR(ind)	3.846893281	4.844749089	4.496366291	3.846893281	4.496366291	9.344330576
Critical value	3.841458821	3.841458821	3.841458821	3.841458821	3.841458821	3.841458821
Conclusion	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
93%						
LR(ind)	8.241998121	8.344533042	6.554636641	8.241998121	7.941979673	9.9872333
Critical value	3.283020287	3.283020287	3.283020287	3.283020287	3.283020287	3.283020287
Conclusion	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
90%						
LR(ind)	11.38740597	9.664159005	9.782766691	11.38740597	9.782766691	9.829641812
Critical value	2.705543454	2.705543454	2.705543454	2.705543454	2.705543454	2.705543454

	SMA- Independence Test			EWMA- Independence Test		
	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation
Conclusion	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected

At the 99% confidence level, we have P(1,1) = 0 and thus do not need to do the independence test. For the other confidence levels, there are only the results generated from VaR models at the 97.5% confidence level, which is accepted, whereas the others are all rejected given the independence test. There is still one result of EWMA-MCS at 99% being accepted. In general, the most valid range of this test would be at 97.5% confidence level.

4.3.4. Joint test

The value of joint test would be the sum value of POF test and independence test. Accordingly, we have the results as follows:

Table 10

Joint test's results

	SMA-Joint Test			EWMA- Joint Test		
	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation	Historical Simulation	Variance-Co-variance	Monte Carlo Simulation
99%						
LR(cc)	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	15.71478321
Critical value						9.210340372
Conclusion	No P(1.1), no independence test --> use POF			No P(1.1), no independence test --> use POF		Rejected
97.5%						
LR(cc)	0.160376377	4.006594147	6.839944016	0.160376377	6.839944016	10.26747227
Critical value	7.377758908	7.377758908	7.377758908	7.377758908	7.377758908	7.377758908
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Accepted
95%						
LR(cc)	3.876511674	4.912650287	4.525984685	3.876511674	4.525984685	9.700684519
Critical value	5.991464547	5.991464547	5.991464547	5.991464547	5.991464547	5.991464547
Conclusion	Accepted	Accepted	Accepted	Accepted	Accepted	Rejected

93%						
LR(cc)	8.503356453	12.05531057	8.176283831	8.503356453	9.252682673	10.24859163
Critical value	5.318520074	5.318520074	5.318520074	5.318520074	5.318520074	5.318520074
Conclusion	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected
90%						
LR(cc)	11.39134989	16.82544427	14.73511649	11.39134989	15.24632802	11.09855491
Critical value	4.605170186	4.605170186	4.605170186	4.605170186	4.605170186	4.605170186
Conclusion	Rejected	Rejected	Rejected	Rejected	Rejected	Rejected

The results of Table 10 show the acceptance range running from 95% to 97.5% confidence levels of all models, which indicates that the VaR model results satisfy both the suitable frequency and the accepted level of independence. However, these joint test results may raise some concerns because even if the results are accepted (rejected), it may not be true that the models' results satisfy (do not satisfy) both POF and independence tests. Katsenga (2013) and Campbell (2005) argued that many of the previous studies also have the same problem when it is possible for the model to pass the joint test but still violate either POF test or independence test, or may even pass those two tests but still violate the joint test. In our study at the 95% confidence level, the join test is accepted, but the models violate the independence test. For the EWMA–Monte Carlo simulation, the joint test is accepted, but the POF test is rejected at the 97.5% confidence level. The reason is that when we do not separate the joint test into POF and independence tests, it is impossible to know which test is accepted. Typically, as the joint test works with chi square of 2 degrees of freedom, while POF and independence tests work with chi square of 1 degree of freedom, there are 2 concerns as the critical value of 2 degrees of freedom does not double the critical value of 1 degree of freedom for 90%–99% confidence levels:

Either of the tests (POF or independence) would be violated, and still the models pass the joint test. This is due to the fact that one of the two tests' critical value is significantly small, compared to the relative critical value of 1 degree of freedom; hence, when the other critical value is only a little greater than the critical value, the join test still lays out an accepted conclusion. For example, at the 95% confidence level, if the POF's ratio is 0.032, which is accepted, and the independence test's ratio is 3.94, which is rejected, then the sum of these two values, which features the join test, would be 3.972 and still accepted.

Another case is that both POF and independence tests are accepted, but the relative joint test is rejected. This happens when both statistical ratio value of both test is merely below the relative critical value of 1 degree of freedom. For example, at the 95% confidence level, the statistical ratio value of POF is 3.2 (accepted) and of the independence test is 3.5 (accepted), but the sum of these tests is 6.7, thereby leading to the rejection.

Table 11 reports the chi-squared critical values of 1 and 2 degrees of freedom.

Table 11

Critical values of Chi-square of 1&2 dof

	1 degree of freedom	2 degrees of freedom
99%	6.634896601	9.210340372
97.50%	5.023886187	7.377758908
95%	3.841458821	5.991464547
93%	3.283020287	5.318520074
90%	2.705543454	4.605170186

In short, one test alone could not be so convincing to prove the validity of the models' results, so we should look at all back-tests to see the most accepted range for the application of VaR models.

4.4. Discussion

Concerning all the back-tests, excluding the Tuff test, we can see at the 97.5% confidence level, the results of three VaR models from 2 volatility-methods are positively supported and accepted by most of the back-tests, including POF, independence, and join tests. Thus, the 97.5% confidence level could be the most valid range of the models' application in VaR determination.

Among these three models, each one has its own pros and cons, and they all have the similar accepted range in both types of tests, implying that the power of these models would be hard to differentiate. Nevertheless, in the context of market condition which is not truly normal, the historical simulation and Monte Carlo simulation are more preferable. Adopting the appropriate model also depends on investors' perspective; for our point of view, the Monte Carlo simulation would be the best due to its flexibility in generating additional observations that capture the recent historical events and the ability to adjust a number of simulations to create market data distribution.

5. Conclusion

First, this study investigates and applies 3 different models to the estimation of the

VaR amounts of VN-30 portfolio throughout the study timeframe in order to check the differences among their results and whether they are significant. It has been found that at the range close to 99% and 90%, the differences become larger while the range of 93%, 95%, and 97.5% reflect small differences.

Next, two types of tests: unconditional and conditional coverage tests, including totally four back-tests, have been conducted to examine the validity of the VaR models in the frequency consistency and independence levels. The Tuff test, as indicated, is not appropriate while the other back-tests show that the most valid range of application is 97.5% confidence level. Therefore, it is recommended that investors find VaR at 97.5% confidence level to enhance the validity power of the models.

In addition, Monte Carlo simulation is the most preferable method suggested in the context of the market condition which is not truly normal due to its flexibility in generating observations based on users' viewpoint.

Finally, the study provides an insight into VaR and 3 basic models applied to VN-30 stock basket in Ho Chi Minh Stock Exchange to see the potential risk of loss that historical basket's movements would likely present and the validity of VaR measurements based on the study assumptions. Hence, it is important that other methods be investigated to develop one's own optimal techniques or extend the models introduced to many other individual portfolios and stock exchanges such as HNX, HNX-30, etc.

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